

EFFECTIVE QFT AND WHAT IT TELLS US ABOUT DYNAMICAL TORSION

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The covariantly constant spacetime torsion is one of the fields which may break Lorentz and CPT symmetry. We review the previous works on the dynamical torsion in the framework of effective quantum field theory (QFT). It turns out that the existence of propagating torsion is strongly restricted by the QFT principles. In particular, the torsion mass must be much greater than the masses of all fermionic particles. In this situation, the main chance to observe torsion is due to some symmetry breaking which may, in principle, produce almost constant background torsion field.

1. Introductory note

In this contribution to the CPT'10 Proceedings I decided to review our papers devoted to the effective QFT approach to the problem of dynamical torsion. The content is not original and is based on Refs. 1,2 (see also Ref. 3), but the purpose is to present it in a maximally simple and qualitative form.

2. Classical torsion gravity and renormalization

An extended version of QED with the general Lorentz and CPT symmetry breaking terms is⁴

$$S = \int d^4x \sqrt{-g} \left\{ \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} D_\mu^\star \bar{\psi} \Gamma^\mu \psi - \bar{\psi} M \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} + \frac{1}{2} (k_{AF})^\rho \varepsilon_{\rho\lambda\mu\nu} A^\lambda F^{\mu\nu} \right\}. \quad (1)$$

Here

$$D_\mu = \nabla_\mu + i q A_\mu; \quad \Gamma^\nu = \gamma^\nu + \Gamma_1^\nu; \quad M = m + M_1, \quad (2)$$

and the quantities Γ_1^ν and M_1 are given by

$$\begin{aligned}\Gamma_1^\nu &= c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + i f^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}, \\ M_1 &= a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + i m_5\gamma_5 + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}.\end{aligned}\quad (3)$$

One of the most phenomenologically important terms is the one which includes $b_\mu = \eta S_\mu$, where S_μ is dual to the spacetime torsion.

In a spacetime with independent metric $g_{\mu\nu}$ and torsion $T_{\beta\gamma}^\alpha$ the connection is nonsymmetric, $\tilde{\Gamma}_{\beta\gamma}^\alpha - \tilde{\Gamma}_{\gamma\beta}^\alpha = T_{\beta\gamma}^\alpha$. Using the metricity condition $\tilde{\nabla}_\mu g_{\alpha\beta} = 0$ we get

$$\tilde{\Gamma}_{\beta\gamma}^\alpha = \Gamma_{\beta\gamma}^\alpha + K_{\beta\gamma}^\alpha, \quad \Gamma_{\beta\gamma}^\alpha = \left\{ \begin{matrix} \alpha \\ \beta\gamma \end{matrix} \right\}, \quad K_{\beta\gamma}^\alpha = \frac{1}{2}(T_{\beta\gamma}^\alpha - T_{\beta\gamma}^\alpha - T_{\gamma\beta}^\alpha). \quad (4)$$

It is convenient to divide torsion into irreducible components

$$T_\beta = T_{\beta\alpha}^\alpha, \quad S^\nu = \varepsilon^{\alpha\beta\mu\nu}T_{\alpha\beta\mu}, \quad q_{\beta\gamma}^\alpha, \quad \text{where } q_{\beta\alpha}^\alpha = \varepsilon^{\alpha\beta\mu\nu}q_{\alpha\beta\mu} = 0. \quad (5)$$

The action of a spinor field minimally coupled to torsion is

$$\begin{aligned}S &= \int d^4x e \left\{ \frac{i}{2}\bar{\psi}\gamma^\mu\tilde{\nabla}_\mu\psi - \frac{i}{2}\tilde{\nabla}_\mu\bar{\psi}\gamma^\mu\psi + m\bar{\psi}\psi \right\} \\ &= \int d^4x e \left\{ i\bar{\psi}\gamma^\mu(\nabla_\mu + \frac{i}{8}\gamma_5 S_\mu)\psi + m\bar{\psi}\psi \right\},\end{aligned}\quad (6)$$

where ∇_μ is the Riemannian covariant derivative (without torsion).

Let us briefly consider renormalization of a gauge QFT in a spacetime with torsion.⁵ In order to achieve consistent renormalizable theory, one has to introduce interaction of matter fields with metric and torsion, plus a vacuum action. With scalars φ torsion interacts only nonminimally,

$$S_{sc} = \int d^4x \sqrt{-g} \left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi_i P_i \varphi^2 \right), \quad (7)$$

where ξ_i are nonminimal parameters and the relevant invariants are

$$P_1 = R, \quad P_2 = \nabla_\alpha T^\alpha, \quad P_3 = T_\alpha^2, \quad P_4 = S_\alpha^2, \quad P_5 = q_{\alpha\beta\gamma}^2. \quad (8)$$

We assume that gauge vector fields do not interact with torsion, since such interaction, generally, contradicts gauge invariance. On top of this we need a vacuum (metric and torsion-dependent) action, which is, in general, rather complicated.⁶ As far as we included all torsion-dependent terms which can be met in the counterterms, the theory described above is renormalizable.

It is important that the spinor field interacts with torsion nonminimally,

$$S = \int d^4x \sqrt{-g} \{ i\bar{\psi}\gamma^\mu (\nabla_\mu + i\eta\gamma_5 S_\mu + i\eta_2 T_\mu) \psi + m\bar{\psi}\psi \}. \quad (9)$$

The minimal interaction corresponds to $\eta = 1/8$, $\eta_2 = 0$.

The β -function for the nonminimal parameter η has universal form,^{5,7} $\beta_\eta = Ch^2\eta$, where h is the Yukawa coupling and the model-dependent coefficient C is always positive. It is easy to see that interaction with background torsion gets stronger in the UV. Moreover, although torsion is a geometric field, the parameters η have to be different for distinct fermions, because within the Standard Model their Yukawa couplings have different values.

3. Dynamical torsion and effective approach

What is the simplest possible action describing dynamical torsion? For the sake of simplicity, consider antisymmetric torsion and a flat metric. Let us try to construct the effective QFT for dynamical (propagating) torsion. The consistency conditions include unitarity of the S -matrix and gauge-invariant but not power-counting renormalizability. A particularly important aspect is that we can neglect possible higher derivative terms. Application of the same approach to quantum gravity⁸ led to some interesting results, although its technical realization remains doubtful.⁹

Let us start from the action of the fermion field

$$S_{1/2} = i \int d^4x \bar{\psi} [\gamma^\alpha (\partial_\alpha - ieA_\alpha + i\eta\gamma_5 S_\alpha) - im] \psi. \quad (10)$$

There are two gauge symmetries, and the second one is softly broken:

$$\begin{aligned} \psi' &= \psi e^{\alpha(x)}, & \bar{\psi}' &= \bar{\psi} e^{-\alpha(x)}, & A'_\mu &= A_\mu - e^{-1} \partial_\mu \alpha(x); \\ \psi' &= \psi e^{\gamma_5 \beta(x)}, & \bar{\psi}' &= \bar{\psi} e^{\gamma_5 \beta(x)}, & S'_\mu &= S_\mu - \eta^{-1} \partial_\mu \beta(x). \end{aligned} \quad (11)$$

These symmetries lead to the form of the torsion action

$$S_{tor} = \int d^4x \{ -a S_{\mu\nu} S^{\mu\nu} + b (\partial_\mu S^\mu)^2 + M_{ts}^2 S_\mu S^\mu \}, \quad (12)$$

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$. In the unitary theory the longitudinal and transverse modes can not propagate simultaneously. Hence, one has to choose one of the parameters a, b to be zero. Indeed, the only correct choice is $b = 0$, because (11) holds in the renormalization of the massless sector.

Thus the torsion action is given by¹

$$S_{tor} = \int d^4x \left\{ -\frac{1}{4} S_{\mu\nu} S^{\mu\nu} + M_{ts}^2 S_\mu S^\mu \right\}. \quad (13)$$

The phenomenological analysis shows the torsion parameters M_{ts} and η can be strongly restricted.^{10,a} However, torsion still can be used, e.g., as an alternative to technicolor models.¹²

The most important question is whether it is really possible to preserve unitarity at the quantum level. The answer to this question is negative. Even in QED, without scalar fields, the one-loop calculations show that $(\bar{\psi}\psi)^2$ -type counterterms are necessary at the one-loop level.^{2,13} Then, in the second loop order we meet a longitudinal divergence for the torsion axial vector, $\int d^4x (\partial_\alpha S^\alpha)^2$.

The violation of unitarity in the fermionic sector occurs when the following condition is not fulfilled:²

$$\eta \frac{m_{fermion}}{M_{ts}} \ll 1. \quad (14)$$

As far as this constraint must hold for all fermions of the MSM, the torsion mass must of the order of, at least, a few TeV, or the coupling η should be extremely small. In both cases, there is no real chance to observe propagating torsion.

4. Conclusions

We have shown, as a result of some complicated calculations in Ref. 10 and especially in Ref. 2, that the existence of the propagating torsion does contradict QFT principles, even in the weakest possible effective version. This result can be seen from two different viewpoints. First, it means that we can restrict the spacetime geometry using effective QFT arguments. As far as we know, this is a unique known example of this sort. On the other hand, the unique ‘chance of survival’ for torsion is to show up as a background field $b_\mu = \eta S_\mu$. The possible mechanisms for the background torsion from symmetry breaking in gravity have been discussed in Ref. 3 and Ref. 14. So, the search of torsion is essentially equivalent to looking for a b_μ term in QED (1) and its extensions in other sectors of the Standard Model. An interesting discussion of possible extensions and last observational constraints has been given in Ref. 15.

^aEarlier, the restrictions for the parameters of purely longitudinal axial torsion vector were established in Ref. 11.

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